# Abstract

This paper examines the statistics of time-series data analysis for hypothetical pricing of futures contracts of the e-mini Dow Jones Industrial Index. The price of the contracts obtained by taking the most recent Dow Jones closing value and applying cumulative random values from the initial closing data. For the pricing of the futures the valuation model used was based on the random walk hypothesis to simplify calculations and as an assumption for long-term holding of the future contracts. We use these predicted prices to calculate weighted values of long and short futures contracts and predict the optimal date to roll the futures contract into the next quarter. Our findings show the optimal time to roll over a Dow Jones futures contract is between 3 and 4 days before the contract expires

# Introduction

Futures are legally binding contracts that require someone to buy or sell an asset at a predetermined date known as the expiry date. The futures market was originally setup to trade contracts on commodities such as grains, metals, meats, indices, and energy, as a means of producers and buyers of these commodities to offset the possible risk of price fluctuations. Depending on the asset, the future is written for a set size of that asset to be exchanged at a future date, so for example oats are traded in lots of 5,000-bushels, feeder cattle in 50,000-pound lots, and Dow Jones Industrial e-minis in 5-time multiples of the Dow Jones Industrial Index. Beyond the use of futures contracts for controlling risk, many people invest in futures contracts as a means of investment and speculation. As part of going long (buying) or short (selling) on a futures contract is the expectation that an asset will be delivered at the expiration date in which the selling party is obligated to deliver the asset and the buying party is expected to take delivery. However, a majority of futures contracts will never be taken delivery of, so long-term investors in futures will want to close out their contract before the expiration date or roll their contracts forward should they want to keep the investment.

Rolling contracts forward involves the process of closing the front (soonest) month and opening a contract in the back (later) month so as to avoid taking delivery of the asset. Rolling forward is done so that investors may invest in futures contracts with the expectation that their position will net them a profit over time, regardless if they are going long or short. To analyze the process of futures contract rolling we will create a set of simulated returns based on the current value of the Dow Jones composite and using the random walk method to create possible values for Dow Jones e-mini contracts. We will set two the expiration months of December, 2018 and March 2019 for the e-mini Dow Jones proposed contracts and see how we would transition from the front month to the back month, before the expiration date. It is imperative that any expiring contracts are rolled over before the expiration date because there are significant financial penalties for failing to do so.

After setting the values of the weights for the two contracts to transition from the front month to the back month we will visually compare the returns to note any significant differences in the expected returns. One thing to note is that this is a simplified model that does not account for many of the possible losses that can be accrued from trading including transaction fees, taxes, and a risk premium, the fact that contracts for further months will almost always be more expensive than for recent months. [1]

# Background

The method we will be using in evaluating the two different future prices for the contracts will be based on the random walk hypothesis. The random walk hypothesis is based off of the efficient market hypothesis or that all known information is built into the price of a security, so any attempt to trade off of this knowledge will not result in any higher returns since all participants in the market already have this knowledge. The efficient market hypothesis is often contrasted with two common trading methodologies, technical analysis and fundamental analysis. Technical analysis, also known as charting, uses different methods of displaying previous trading data including price movement, volume, and acceleration of purchasing to show patterns that could indicate an opportune time to buy or sell an asset. Technical analysis is contrasted with fundamental analysis which uses the underlying financial, business, and economic data associated with a financial asset to determine the correct valuation. The efficient market hypothesis is used in contrast to these two methods of valuation to state that any possible rational valuation obtained from either technical or fundamental analysis is already built into the price of the asset. The valuation of assets can change over time as new information is made available to the trading public, but in general all available information is already taken into account by the current asset price. [2]

The random walk hypothesis builds off of the efficient market idea, as detailed by Fama, because of how new information available to the market is instantaneously built into the price of the asset. This instantaneous correction has two effects, many times the price will overcorrect or under correct to this new information followed some time later to a correction to this mis valuation, in a seemingly random fashion. Simply put the random walk method means that the future movement of an asset is independent of the current price. [2]

# Method

We start by retrieving the closing prices of the Dow Jones Industrial Average from Yahoo! Finance using the pandas-datareader API. We are using the recently released version 0.7.0 which contains fixes to correctly read Yahoo! Finance market data. This API accepts a stock symbol and optionally a start and end date. We are retrieving data from Jan 2010 up to present day. Table 1 below shows the first five data points retrieved, and last five data points retrieved.

Table - Dow Jones Closing Prices

|  |  |
| --- | --- |
| Date | ^DJI Closing Price |
| 2010-01-04 | 10583.960 |
| 2010-01-05 | 10572.020 |
| 2010-01-06 | 10553.680 |
| 2010-01-07 | 10606.860 |
| 2010-01-08 | 10618.190 |
| … | … |
| 2018-09-17 | 26062.119 |
| 2018-09-18 | 26246.960 |
| 2018-09-19 | 26405.760 |
| 2018-09-20 | 26656.980 |
| 2018-09-21 | 26743.500 |

As explained above we are interested in comparing the values of two e-mini Dow futures contracts, the front contract expiring on December 21, 2018, and the back contract expiring on March 15, 2019.

We want to use the random walk method to simulate future values for each contract. A series is created of ransom floating points between -25 and + 25. This series is the turned into a cumulative series, meaning each value is added to the value before. Although the change in value is random (using this random number series), the absolute value of the current short contract is based on what the previous price was such the where t is the index for today, *sct* is equal to today’s value of the short contract, *sct-*1 is equal to price of the short contract on the previous business day and (*walk value*)t is the random value for today’s index. This chart shows a run of positive numbers early in the index, but then a run of negative numbers bringing the cumulative total down as we expect in a random distribution of positive and negative integers Figure 1 below shows what this cumulative random value looks like.

Figure - Random Walk Cumulative Value

A close up of text on a white background

Description generated with very high confidence

A similar series of random integers is created ranging from -2.5 to +2.5. These random numbers are added to the short contract value to come up with the long contract value such that where *lc*t is equal to today’s value of the long contract, lct-1 is equal to the price of the long contract on the previous business day is equal to the random value for today’s index and is equal to the random value used to add variance to the long contract. This chart does not show the same type of run we saw in the walk chart, but still is the randomness we expect is evident. This chart is seen below in Figure 2.

Figure - Perturb random value

A close up of a logo

Description generated with very high confidence

Using the equations for short contracts,which has stock ticker symbol of YMZ18, and the long contract, which has a stock ticker symbol of YMH19, we create a series of prices for each of the contracts. The table below show a sample of the prices for each contract from October 29, 2018 to November 9, 2018.

Table - Future Contract Prices

|  |  |  |
| --- | --- | --- |
| Date | YMZ18 | YMH19 |
| 2018-10-29 | 26966.75 | 26968.75 |
| 2018-10-30 | 26953.25 | 26951.50 |
| 2018-10-31 | 26939.75 | 26939.75 |
| 2018-11-01 | 26931.00 | 26931.50 |
| 2018-11-02 | 26906.75 | 26906.75 |
| 2018-11-05 | 26930.75 | 26931.00 |
| 2018-11-06 | 26955.00 | 26957.00 |
| 2018-11-07 | 26961.00 | 26960.25 |
| 2018-11-08 | 26955.50 | 26953.50 |
| 2018-11-09 | 26931.50 | 26931.75 |

Now that we have a series of values for each contract, we want to assign a weighted value for each contract as time moves forward. In other words, which contract has more value to be owned on a specific date. When using a rolling contracts strategy, ideally we want to sell the short contract close to the same price as we buy the long contract thereby maintaining our position rolling forward. A simple algorithm weights the value of each contract as the expiration date of the short contract approaches. The weight for a fully valued contract has a value of ‘1’. As the expiration date approaches the weighted value of the short contract goes down toward zero as the weighted value of the long contract goes toward one. Figure 3 shows where the cross-over of values happens.

Figure - Weighted return cross over point

A screenshot of a cell phone

Description generated with high confidence

Table - Weights of contracts

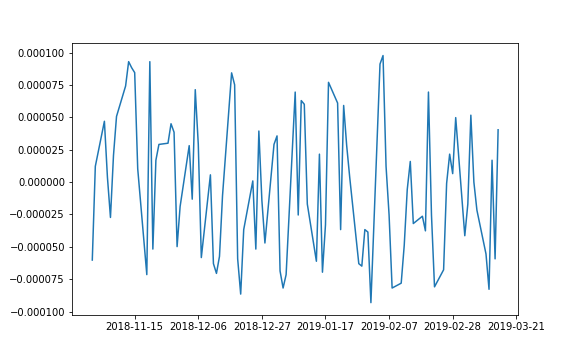
|  |  |  |
| --- | --- | --- |
| Date | YMZ18 | YMH19 |
| 2018-12-12 | 1.0 | 0.0 |
| 2018-12-13 | 1.0 | 0.0 |
| 2018-12-14 | 0.8 | 0.2 |
| 2018-12-17 | 0.6 | 0.4 |
| 2018-12-18 | 0.4 | 0.6 |
| 2018-12-19 | 0.2 | 0.8 |
| 2018-12-20 | 0.0 | 1.0 |
| 2018-12-21 | 0.0 | 1.0 |

# Results

The random walk model used to predict the prices of the short and long futures contracts of the Dow Jones provided a useful, but very simplified model. The model attempts to predict when we should rollover a short contract to the next quarter. This model shows futures contracts should be rolled over no later than 4 days before the contracts expire in order not to lose money in the rollover process. Trying to rollover the futures any earlier than 6 days before the contract expires exposes us to a risk we are paying more for the rollover contract than perhaps we need to. Waiting until the last 3 days before the contract expires exposes us to a risk we cannot sell the short contract for enough money and risk either losing money during the roll over, or even worse, having to take delivery of the stock which may put us in a position of having to burn capital.

As shown in Figure you can see returns are kept consistent while switching the balance of the portfolio from the front contract to the back contract. The primary goal of rolling the contract over is to reduce the risk premium paid, while holding long-term position in the futures contract.

Figure - Rate of returns through cross over point



# Future Work

In future research there are two means of improve any predicted returns from using this model, better factoring of costs for trading from a front contract to a back contract and improved random walk modeling. As mentioned earlier, this simplified model does not account for many of the transactional fees, taxes and risk premiums associated with rolling contracts forward. While this simple model is useful to demonstrate future rolling contracts in principle, it does not properly show the effect on returns that rolling futures forward would have in the real world. Once these additional costs are factored it may be more beneficial to purchase contracts that are more than 3 months from expiring, this would increase the time between having to roll future contracts forward again. Purchasing contracts that are further out can pose problems though, if the volume is low for the further expiry month is lightly traded. Low trading volume can lead to a bigger difference between the expected trade price and the price at execution, also known as the bid-ask spread. Proper modeling of these additional fees would help to better model the proper time of rolling your contracts and how far out the expiration date of the new contracts should be.

Future models could use an improved method of asset valuation beyond random walk. There is increasing research that random walk does not hold for all circumstances and even some flat out rejects the random walk hypothesis, citing that long swings in exchange rate data are not adequately explained by the random walk methodology. [3] Some research does not fully reject the random walk hypothesis, rather that it is only effective for explaining the movement of certain markets under certain circumstances. As demonstrated by Chuluun, Eun, and Kilic (2009) they found that assets with higher investment intensity, or greater participation in the asset’s market by speculators and investors as opposed to producers and consumers, were less likely to reject the random walk hypothesis. [4]

The general assumption for this exercise is that all relevant information is built into the price of the asset, but if this assumption is incorrect then the general assumption of holding the asset past the front contract’s expiry date can be dropped. Another solution of closing the contract would need to be considered and an appropriate model should be created that takes this into account.

# References

|  |  |
| --- | --- |
| [1] | A. Kamara, "The Behavior of Futures Prices: A Review of Theory and Evidence," *Financial Analysts Journal,* vol. 40, no. 4, pp. 68-75, 1984. |
| [2] | E. F. Fama, "Random Walks in Stock Market Prices," *Financial Analysts Journal,* vol. 21, pp. 55-59, 1965. |
| [3] | F. Klaassen, "Long swings in exchange rates: Are they really in the data?," *Journal of Business & Economic Studies,* vol. 23, pp. 87-95, 2005. |
| [4] | T. Chuluun, C. S. Eun and R. Kiliç, "Investment intensity of currencies and the random walk hypothesis: Cross-currency evidence," *Journal of Banking and Finance,* vol. 35, pp. 372-387, 2011. |

# Appendix

Python Code

import datetime

import pandas as pd

import numpy as np

import matplotlib.pyplot as plt

from pandas\_datareader import data as web

pd.options.display.max\_rows = 10

px = web.get\_data\_yahoo('^DJI')['Adj Close']

from datetime import datetime

expiry = {'YMZ18': datetime(2018, 12, 21),

'YMH19': datetime(2019, 3, 15)}

expiry = pd.Series(expiry).sort\_values()

np.random.seed(12347)

N = 200

walk = (np.random.randint(0, 200, size=N) - 100) \* 0.25

perturb = (np.random.randint(0, 20, size=N) - 10) \* 0.25

walk = walk.cumsum()

rng = pd.date\_range(px.index[0], periods=len(px) + N, freq='B')

near = np.concatenate([px.values, px.values[-1] + walk])

far = np.concatenate([px.values, px.values[-1] + walk + perturb])

prices = pd.DataFrame({'YMZ18': near, 'YMH19': far}, index=rng)

prices=prices[['YMZ18', 'YMH19']]

prices['2018-10-29': '2018-11-11']

prices['2018-10-29': '2018-11-11']

plt.xlabel ('Day Index')

plt.ylabel ('Cumulative Value')

plt.plot(walk)

plt.savefig('walk.png')

plt.show()

plt.xlabel ('Day Index')

plt.ylabel ('Random value')

plt.plot(perturb)

plt.savefig()

plt.show()

def get\_roll\_weights(start, expiry, items, roll\_periods=5):

# start : first date to compute weighting DataFrame

# expiry : Series of ticker -> expiration dates

# items : sequence of contract names

dates = pd.date\_range(start, expiry[-1], freq='B')

weights = pd.DataFrame(np.zeros((len(dates), len(items))),

index=dates, columns=items)

prev\_date = weights.index[0]

for i, (item, ex\_date) in enumerate(expiry.iteritems()):

if i < len(expiry) - 1:

weights.loc[prev\_date:ex\_date - pd.offsets.BDay(), item] = 1

roll\_rng = pd.date\_range(end=ex\_date - pd.offsets.BDay(),

periods=roll\_periods + 1, freq='B')

decay\_weights = np.linspace(0, 1, roll\_periods + 1)

weights.loc[roll\_rng, item] = 1 - decay\_weights

weights.loc[roll\_rng, expiry.index[i + 1]] = decay\_weights

else:

weights.loc[prev\_date:, item] = 1

prev\_date = ex\_date

return weights

weights = get\_roll\_weights('11/01/2018', expiry, prices.columns)

weights.loc['2018-12-12':'2018-12-21'].plot();

display (weights.loc['2018-12-12':'2018-12-21'])

plt.savefig("weights.png")

rolled\_returns = (prices.pct\_change() \* weights).sum(1)

plt.figure(figsize=(8,5))

plt.plot(rolled\_returns.loc['2018-11-1':'2019-3-15']);

plt.savefig("rolled\_returns.png")